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17. ABSTRACT (MAX. 250 WORDS)

Materials are included for both enriched and accelerated mathematics courses, with adaptations and modifications to meet the needs of most mathematics classes. Unit and time plans are given for both semesters and textbooks to which the guide is keyed are listed. Topics, objectives, suggested teaching techniques, available films and film strips, evaluation techniques, supplementary references and vocabulary development are given for each unit. For easier location, vocabulary is also listed in alphabetical order. Film and filmstrip titles should be useful to any teacher seeking audio-visual materials in mathematics.

Units include: sets of numbers; variables, open expressions and open sentences; equations and inequations; systems of equations and inequations, products and factors, algebraic fractions; ratio, proportions and variation; roots, powers and radicals; quadradic equations and inequations and numerical trigonometry.

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TEACHERS' GUIDE

Eighth Grade Algebra for the Academically Talented

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U. S. DEPARTMENT OF HEALTH, EDUCATION AND WELFARE
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Issued by
E. E. HOLT
Superintendent of Public Instruction
Columbus, Ohio
1963

TEACHERS' GUIDE

Eighth Grade Algebra for the Academically Talented

Prepared by

CLEVELAND PUBLIC SCHOOLS

Division of Mathematics

In Cooperation With

THE OHIO DEPARTMENT OF EDUCATION

Under the Direction of

R. A. HORN

Director, Division of Special Education

Columbus, Ohio

1963

ACKNOWLEDGMENTS

The Division of Special Education is particularly grateful to the members of the Mathematics Curriculum Committee of the Cleveland Public Schools for their contributions to this publication. The committee was composed of:

Dr. William B. Levenson, Superintendent

Mr. Alva R. Dittrick, Deputy Superintendent

Dr. Harry E. Ritchie, Assistant Superintendent

Mrs. Dorothy E. Norris, Directing Supervisor of Major Work

Mr. Lawrence Hyman, Directing Supervisor of Mathematics

Mr. Martin H. Preuss, Supervisor

Mr. James A. Clement, Coordinator

Miss Mary T. Gavin, Carl F. Shuler Junior High School

Miss E. Jean Thom, Wilbur Wright Junior High School

To each of these people, we offer our sincere thanks and appreciation. We feel that through their efforts a valuable addition has been made in the enrichment and acceleration of junior high school mathematics in Ohio.

THOMAS M. STEPHENS
Administrative Assistant
Division of Special Education

R. A. HORN
Director
Division of Special Education

FOREWORD

During the past three years, the Cleveland Public Schools in cooperation with the Ohio Department of Education have conducted a demonstration project in junior high school mathematics. This demonstration project has been supported by funds that were appropriated through legislative action.

This junior high school mathematics program was designed to augment the horizontal enrichment of the mathematics courses with vertical acceleration of course content. The teachers' guides were developed for the seventh, eighth, and ninth grade mathematics courses by the teachers and supervisors participating in the project.

This teachers' guide is an outgrowth of that demonstration project and is presented to the educators of Ohio as part of our continued efforts to provide for the school children of Ohio. It is my hope that the schools of Ohio will be able to modify or adopt this guide to meet their needs in the area of mathematics.

E. E. HOLT
Superintendent of Public Instruction

INTRODUCTION

Background

In September, 1960, the Cleveland Public Schools in cooperation with the Ohio Department of Education, Division of Special Education began a demonstration project in junior high school mathematics for academically talented students. This mathematics demonstration project was designed to go beyond homogeneous grouping and classroom enrichment. An accelerated program was begun in the seventh grade by combining the seventh and eighth grade programs into one year. Algebra I and II was introduced to these academically talented students in the eighth grade, and a combined plane and solid geometry course was introduced in the ninth grade. By accelerating the academically talented students in the junior high school, the opportunity to take an additional three semesters of college preparatory mathematics would be available to these students in the high school.

After three years, the Cleveland Public Schools have had an opportunity to evaluate the program both objectively and subjectively. The objective evaluation has been done through the use of various standardized tests given to the accelerates, the best regular classes at each grade level, and the regular classes at each grade level. In almost all cases the accelerates scored as well as or better than the groups with which they were compared on the standardized tests. It should be remembered that the accelerated students are at least six months to one and one-half years younger chronologically than the comparison groups in the regular curriculum.

A subjective evaluation was made by questioning administrators, counselors, teachers, and pupils. The general concensus of opinion of these people was that the program has been highly successful and should remain a part of the junior high school curriculum, however it should be sufficiently flexible to meet the changing needs of the school and the pupils involved.

Suggestions for Using the Guide

This teachers' guide contains materials for both enriched and accelerated mathematics courses. The materials are presented in such a way that they can be easily adapted or modified to meet the needs of most mathematics classes. The suggestions and supplementary references found in each unit should be a valuable aid to the mathematics teacher, in a regular or accelerated class.

The film and filmstrips are coded for the Cleveland Public School system, but the titles should be useful to the teacher who is seeking audio-visual materials in conjunction with a particular unit plan. Some of the evaluative materials are copyrighted tests of the Cleveland Public Schools, but much of the evaluation is a function of the individual teacher or can be adapted from tests found in the basal and supplementary text books.

It is our hope that this guide can be adapted or modified to meet the needs of both the experienced and inexperienced teacher and thereby lead to the improvement of the secondary school mathematics program.

ARTHUR R. GIBSON
Education Specialist
Programs for the Gifted

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TEXTBOOKS

This guide has been keyed to the following textbooks. Note that under *References* the pages are for the text designated by the letter heading the column.

A. Mallory, Virgil S., Skeen, Kenneth C., and Meserve, Bruce C., *First Course in Algebra,* Syracuse: The L. S. Singer Co., 1961.

B. Fehr, Howard F., Carnahan, Walter H., and Beberman, Max, *Algebra First Course,* Boston: D. C. Heath and Co., 1962.

C. Aiken, Daymond J., Henderson, Kenneth B., and Pingry, Robert E. *Algebra: Its Big Ideas and Basic Skills,* New York; McGraw Hill Book Co. Inc., 1960.

D. Weeks, Arthur W., and Adkins, Jackson B., *First Course in Algebra,* New York: Ginn and Company, 1961.

E. Dolciani, Mary P., Berman, Simon L., and Freilich, Julius, *Modern Algebra Structure and Method,* Book I., Boston: Houghton Mifflin and Company, 1962.

UNIT AND TIME PLAN

Semester I

Unit		Time
Pre-Unit	Sets and Set Notation	
I.	Sets of Numbers	4 weeks
II.	Variables, Open Expressions and Open Sentences	7 weeks
III.	Equations and Inequations	5 weeks
IV.	Systems of Equations and Inequations	3 weeks

Semester II

V.	Products and Factors	4 weeks
VI.	Algebraic Fractions	5 weeks
VII.	Ratio, Proportion and Variation	2 weeks
VIII.	Roots, Powers and Radicals	2 weeks
IX.	Quadratic Equations and Inequations	3 weeks
X.	Numerical Trigonometry	2 weeks

Pre-Unit - Sets and Set Notation

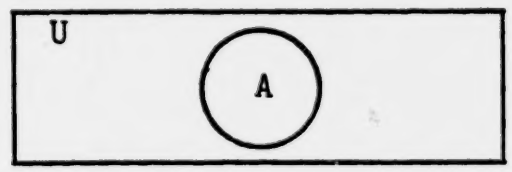
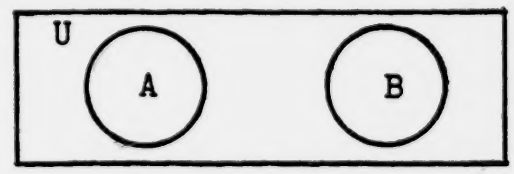
TOPICS	OBJECTIVES	SUGGESTIONS
Describing a Set	To reinforce the understanding of sets and set notation	The concepts, vocabulary, and symbolism of sets developed in Pre-Unit will be used throughout this guide to clarify, simplify, or unify topics covered herein. The past experience of the members of the class should determine the development of both Pre-unit and Unit I.
	To develop the ability to describe a set	Suggested sequence: List expressions which indicate groups or collections of things. A collection of stamps A chain of stores A flock of birds A herd of cattle A set of books Describe and list members (elements) of a set. The set of all the days of the week whose names begin with "T". Tuesday, Thursday The set of all the months of the year whose names begin with "G". None Tabulate, that is, list members of a set within braces. The set of all integers between 2 and 8. {3, 4, 5, 6, 7} The set of all natural numbers. {1, 2, 3, 4, 5,...} Three dots are used to indicate that there is no limit to the number of members a set may have. The set of all natural numbers between 5 and 46. {6, 7, 8, 9, 10,...45} Three dots are used to indicate "and so on up to". The set of all integers less than 4 and at the same time greater than 5. Ø A set having no members is indicated by using the symbol Ø which is read "the empty set" or "the null set". Note: Braces are never used with this symbol.

FILMS AND FILM STRIPS	EVALUATION	SUPPLEMENTARY REFERENCES	VOCABULARY
	List members of a set and have pupils describe the set Describe a set and have pupils tabulate the set Matching test on symbols and vocabulary	"Report of the Commission on Mathematics - Appendices" --College Entrance Examination Board <u>Algebra, First Course</u>, Second Edition --Fehr, Carnahan, Beberman pp. 212-216 <u>Modern Algebra Structure and Method</u> --Dolciani, Berman, Freilich pp. 10-33, 60-61 "Modern Mathematics in the Elementary and Junior High Schools" --Williams pp. 74-93 "Modern Mathematics Introductory Concepts and Their Implications" --Evenson "Modern Mathematics Topics and Problems" --Aiken and Beseman pp. 1-15, 112-119 "Sets, Sentences and Operations" --Johnson and Glenn pp. 2-13, 29 "Supplementary Units in Contemporary Arithmetic and Elementary Algebra" --Stein pp. 32S-34S "Worktext in Modern Mathematics" --Marston pp. 1-12	set set notation member element tabulate braces { } "the set of" Ø "the empty set" or "the null set"

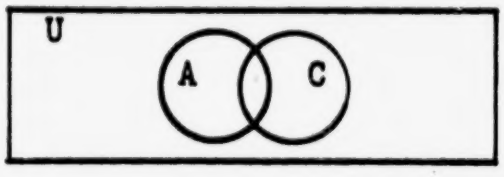
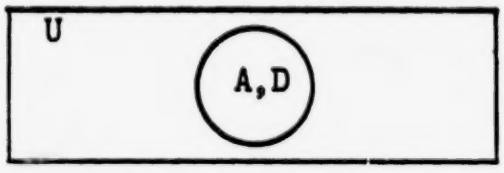
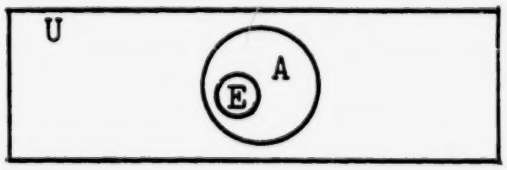
Pre-Unit - Sets and Set Notation

TOPICS	OBJECTIVES	SUGGESTIONS
		Write descriptions of sets. {California, Oregon, Washington, Hawaii, Alaska} The set of all states of the United States bordering on the Pacific Ocean.
Finite and Infinite Sets	To develop an understanding of finite and infinite sets	Finite: The set of natural numbers between 7 and 413. {8, 9, 10, 11,...412} Infinite: The set of all natural numbers. {1, 2, 3, 4, 5,...}
Comparing Sets	To develop the ability to compare one set with another set	Compare the members of one set with the members of a second set.
Unequal Sets		The sets M = {a, c, e} and N = {b, d, f} are different because they do not have the same members. This may be written M ≠ N.
Equal Sets		The sets X = {a, b, c} and Y = {c, a, b} are equal because they have the same members. This may be written X = Y. (Usually a capital letter is used to designate a set.)
Equivalent Sets	To develop an understanding of one-to-one correspondence	Match the members of one set to the members of the second set. A = {d, e, f} ↕ ↕ ↕ B = {x, y, z} Whenever one set has a one-to-one correspondence with a second set, the sets are said to be equivalent. A ⟷ B
Universal Set	To develop an understanding of the meaning of universal set	Whenever we talk about sets we are talking about a collection of things which have something in common. The set of states of the United States. This large set is called the universe or universal set. The symbol for the universal set is "U".

FILMS AND FILM STRIPS	EVALUATION	SUPPLEMENTARY REFERENCES	VOCABULARY
			finite
			infinite
			≠ "is not equal to"
			= "is equal to"
		True, false test	
			one-to-one correspondence
			⟷ "is equivalent to"
			U "universe" or "universal set"

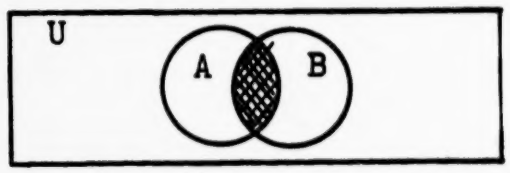
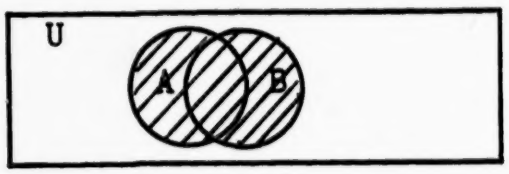
TOPICS	OBJECTIVES	SUGGESTIONS
Subsets	To develop an understanding of the meaning of subsets	Individual members of a universal set are called subsets. Ohio, Michigan, Illinois is a subset of the universal set: The set of states of the United States. By inductive reasoning develop the formula for number of subsets, P, if the number of members of the original set is n. $P = 2^n$
Venn Diagrams	To develop the ability to construct and interpret Venn diagrams	Venn diagrams are used to picture the relationships between sets rather than their comparative size or numbers. ##Venn Diagram Chart Scott, Foresman and Co. 433 East Erie Street Chicago 11, Illinois
Diagramming Universal Sets		U = the set of all pupils in this class.
Diagramming Subsets		A subset of the universal set is represented by points within and on a circle placed within the rectangle. A = the set of all boys in this class. 
Diagramming Disjoint Sets		Two sets may have no members in common. A = the set of all boys in this class. B = the set of all girls in this class.  Set A is disjoint from set B.

FILMS AND FILM STRIPS	EVALUATION	SUPPLEMENTARY REFERENCES	VOCABULARY
	Completion tests List subsets of given universal set		subset Venn diagram
	Draw Venn diagrams to represent relationships		disjoint

TOPICS	OBJECTIVES	SUGGESTIONS
Diagramming Intersecting Sets		Two sets may have one or more members in common. A = the set of all boys in this class. C = the set of all pupils earning an "A" in mathematics in this class.  Set A is said to intersect set C.
Diagramming Equal Sets		Two sets may have all their members in common. A = the set of all boys in this class. D = the set of all pupils in this class who are not girls.  Set A = Set D or A = D
Diagramming Subsets		All members of one set are also members of the other set. A = the set of all boys in this class. E = the set of all boys with blue eyes in this class.  Set E is a subset of set A. °°°Develop Venn diagrams to represent relationships among three or more sets. Write set relationships to fit a given Venn diagram.

FILMS AND FILM STRIPS	EVALUATION	SUPPLEMENTARY REFERENCES	VOCABULARY
			intersect
			equal

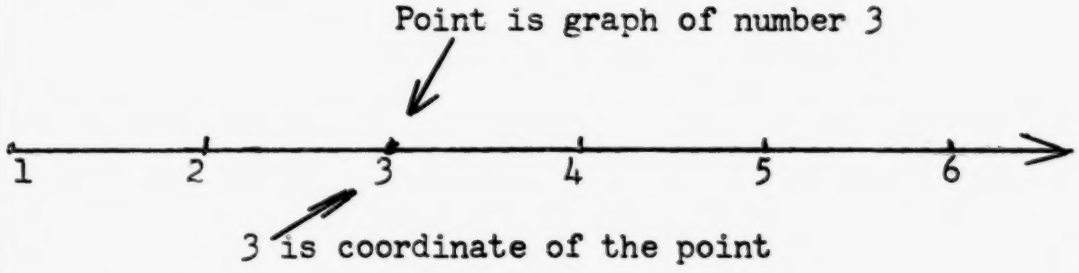
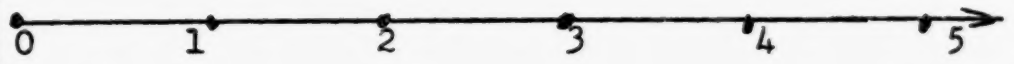
Pre-Unit - Sets and Set Notation

TOPICS	OBJECTIVES	SUGGESTIONS
Operations with Sets Intersection Set	To develop the ability to perform operations with sets	The intersection of two sets refers to the members which both sets have in common. $A = \{a, b, c, d\}$ $B = \{c, d, e, f\}$ The intersection of set A and set B is the set $\{c, d\}$ The symbol used for intersection is $\cap$ and is read "intersection". $A \cap B$ is the symbol <u>for the set</u> of members of set A who are also members of set B. $A \cap B = \{c, d\}$ 
Union Set		The union of two sets refers to all the members of the sets on which the operation is performed. $A = \{a, b, c, d\}$ $B = \{c, d, e, f\}$ The union of set A and set B is the set $\{a, b, c, d, e, f\}$ The symbol used for union is $\cup$ and is read "union". $A \cup B$ is the symbol <u>for the set</u> of all members of A together with all members of B. $A \cup B = \{a, b, c, d, e, f\}$ 

FILMS AND FILM STRIPS	EVALUATION	SUPPLEMENTARY REFERENCES	VOCABULARY
			$\cap$ "intersection"
	Matching tests Solve problems using Venn diagrams		$\cup$ "union"

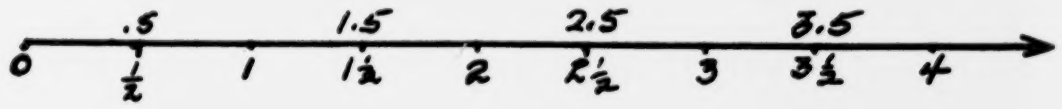
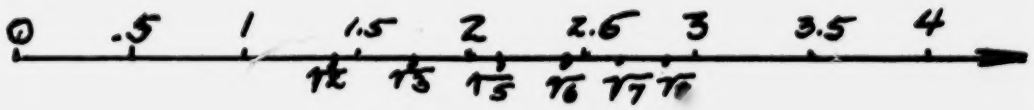
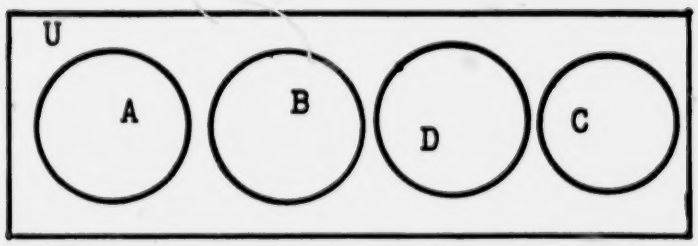
Unit I - Sets of Numbers (4 weeks)

Topics	Objectives	References A	B	C	D	E
Sets of Numbers	To strengthen and further develop an understanding of the sets of numbers comprising the real number system by means of: number lines Venn diagrams tabular classification examples					
Natural Numbers	To develop a more thorough understanding of natural numbers	1-3,19				1-5
Zero	To develop an appreciation of the importance of zero	12-13		55-56	31-32	

Vocabulary	Suggestions
number numeral	Differentiate between number and numeral.
	Mimeographed sheets of number lines will be helpful in developing the following material.
number line arithmetic scale natural number counting number magnitude coordinate point graph	Number line-arithmetic scale Natural (counting) numbers can be arranged in <u>order of magnitude</u> (size) and associated with points on an uniformly scaled line. The number which is associated with the point on the number line is called the <u>coordinate</u> of the point on the line. The point is called the <u>graph</u> of the number. <u>Natural Number Line</u>  Develop the concept of "<u>between-ness</u>". The set of numbers between 2 and 6 = {3, 4, 5} Develop the concept of "<u>inclusiveness</u>". The set of numbers between 2 and 6 inclusive = {2, 3, 4, 5, 6}
zero	<u>Whole Number Line</u> 

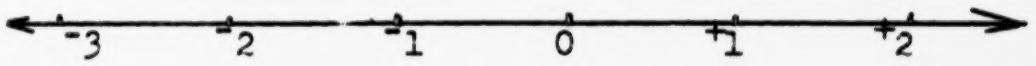
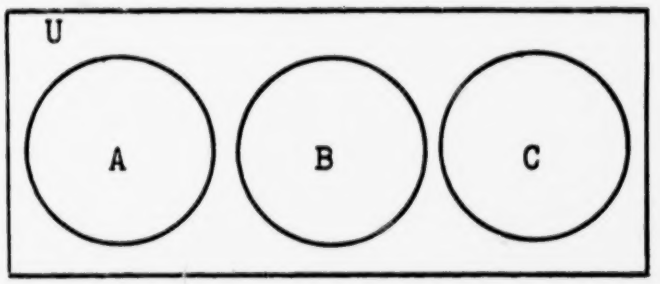
Unit I - Sets of Numbers

Topics	Objectives	References				
		A	B	C	D	E
Fractions Common Decimal	To develop a more thorough understanding of fractions	4-6				398-400

Vocabulary	Suggestions
positive common fraction proper improper simple complex decimal fraction mixed number pure decimal mixed decimal density rational number	Positive Rational Number Line  Develop the concept of density. Between every pair of fractions there is another fraction. Further develop the concept of infinite and finite sets.
irrational number real number	Positive Real Number Line  Venn diagram for set of positive real numbers:  U = set of all positive real numbers (numbers of arithmetic) A = set of all natural numbers B = 0 C = set of all positive fractions D = set of all positive irrational numbers

Unit I - Sets of Numbers

Topics	Objectives	References A	B	C	D	E
Directed Numbers	To develop the ability to understand the meaning of directed numbers	91-96	55-64 84	53-55 56-58	143-147 184-185	111-113 123-124

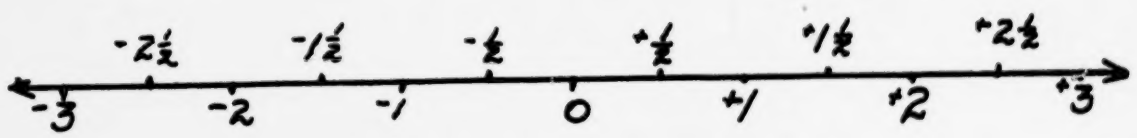
Vocabulary	Suggestions
integer directed number signed number positive number negative number direction distance opposite quality absolute value (magnitude) origin coordinate line vector	Raised signs may be used to indicate directed quantities. $^{+}5$ (positive five) is said to be positively directed or measured in a positive direction. $^{-}5$ (negative five) is said to be negatively directed or measured in a negative direction. Be sure that pupils understand that $^{-}2$ and $^{+}3$ name <u>new numbers</u> and not <u>some operation</u> performed on 2 and 3. Note: Raised signs should be used only until the concept of directed numbers is understood. Integer Number Line:  Venn diagram for set of integers  U = set of all integers A = set of all natural numbers B = { 0 } C = (set of all negatives of natural numbers)

Unit I - Sets of Numbers

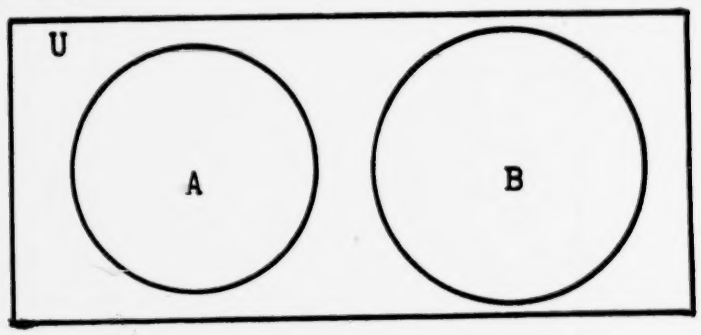
Topics	Objectives	References				
		A	B	C	D	E

Vocabulary	Suggestions

Rational Number Line:

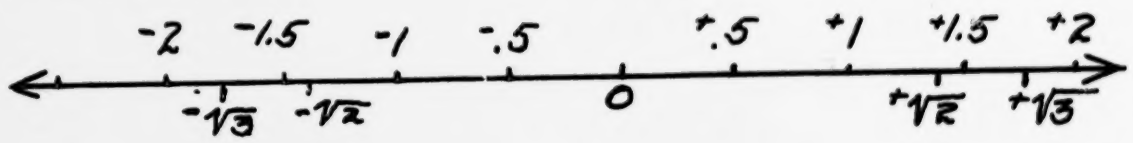


Venn diagram for set of rational numbers

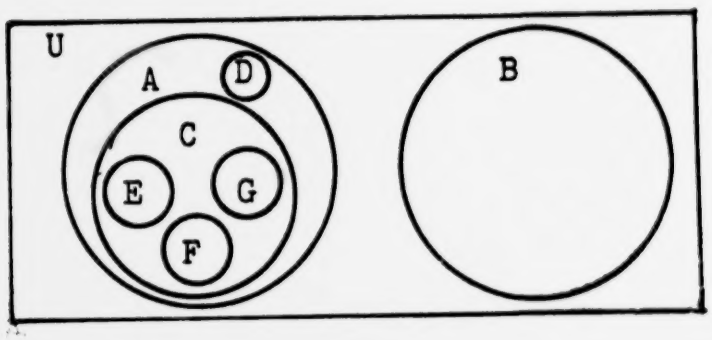


U = set of all rational numbers
A = set of all integers
B = set of all fractions

Real Number Line:



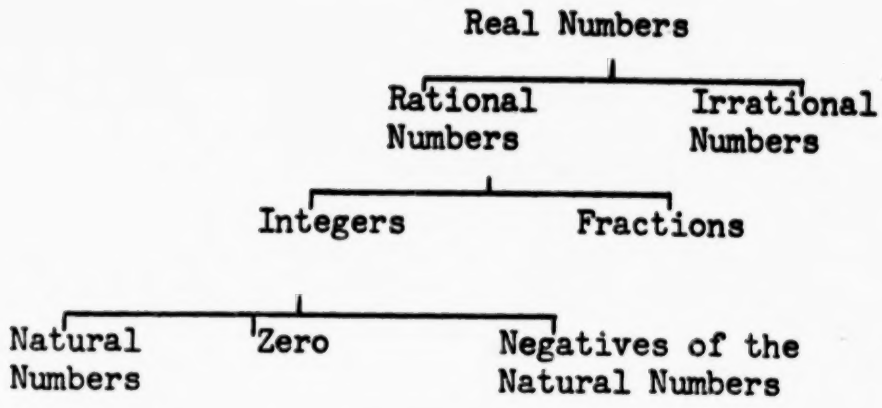
Venn diagram for set of real numbers



U = set of all real numbers
A = set of rational numbers
B = set of all irrational numbers
C = set of all integers
D = set of all fractions
E = set of all natural numbers
F = { 0 }
G = set of all negatives of the natural numbers

Unit I - Sets of Numbers

Topics	Objectives	References				
		A	B	C	D	E
						180 A-F
		157-162	60-63 220-224	178-181	217-219	337-340 358
		°141-157				°354-356 °142-144

Vocabulary	Suggestions
horizontal axis vertical axis abscissa ordinate quadrant ordered pair	Tabular Classification of Numbers:  Other methods for development of directed numbers: Line vectors- B-pp.60-63 Locating points on coordinate paper with relation to horizontal and vertical axes See: Algebra, First Course- - Mayor and Wilcox pp. 16-19 Graphing pictures -- E- p.358 and Mathematics Teacher January,1959, pp. 41-43. Graphing to locate hidden treasure Automobile trips east-west, north-south Football field (gain-loss) Elevator (up-down) Thermometer (rise-fall) See: "Adventures in Graphing" - Glenn and Johnson °Reading, drawing and interpreting graphs should be familiar material to these pupils. A rapid review may be taken if needed. °°°Statistics may be introduced as a supplementary topic at this time. Sound film: How's Chances? Association Films 561 Hillgrove Ave. La Grange, Illinois

Unit I - Sets of Numbers

Topics	Objectives	References A	B	C	D	E
Properties of Numbers	To develop the ability to understand the properties governing numbers	7-9,19	11-14	14,15 67,121 11	11-13 34-35 186	73-75
Commutative Property	To develop the ability to use precise vocabulary and precise symbolism					
Associative Property						
Distributive Property			15-19			19-24 75-77, 78
Identity Elements		12-15 20				77 120-123
					28,29	

Vocabulary	Suggestions
commutative property	
associative property	
distributive property inclusion parentheses ( ) brackets ⊏ ⊐ braces { } vinculum ———	Use symbols of inclusion to avoid ambiguous statements. 3 X 2 + 4 = ? Does this statement mean: (3 X 2) + 4 or 3 X (2 + 4) ?
zero additive identity element	<u>Properties of zero</u>: Let a represent any quantity except zero then, a + 0 = a 0 + a = a a - 0 = a a ∘ 0 = 0 0 ∘ a = 0 0 ÷ a = 0 a ÷ 0 has no meaning
additive inverse	For every directed number ^{+}a there is a unique number ^{-}a such that $(^{+}a) + (^{-}a) = (^{-}a) + (^{+}a) = 0$ Addition of a directed number and its opposite (additive inverse) gives a sum equal to 0. Refer to number line. See: "Modern Mathematics in the Elementary and Junior High Schools" by Williams, Read, Williams-pp.41-57
one multiplicative identity element	<u>Properties of one</u>: Let a represent any quantity except zero then, 1 ∘ a = a a ∘ 1 = a a ÷ 1 = a $1 \div a = \frac{1}{a}$ a ÷ a = 1 $1^{a} = 1$
reciprocal multiplicative inverse	For every quantity "a" there is a unique quantity $\frac{1}{a}$ (reciprocal) such that $a \circ \frac{1}{a} = 1$. Multiplication of a quantity and its reciprocal (multiplicative inverse) gives a product equal to 1.

Unit I - Sets of Numbers

Topics	Objectives	References A	B	C	D	E
Inverse Operations		10-11	96-97	32		81

Vocabulary	Suggestions
inverse	An <u>inverse</u> of an operation is an operation which <u>undoes</u> or <u>annuls</u> a given operation when it follows that given operation. Example: Walking to school and walking home. Putting a jigsaw puzzle together and taking it apart. <u>Addition</u> and <u>Subtraction</u> 10 + 4 = 14 ; 14 - 4 = 10 8 + 9 = 17 ; 17 - 9 = 8 <u>Multiplication</u> and <u>Division</u> 2 X 8 = 16 ; 16 ÷ 8 = 2 5 X 9 = 45 ; 45 ÷ 9 = 5

Unit I - Sets of Numbers

Topics	Objectives	References A	B	C	D	E
Closure						70-73
Fundamental Operations Addition	To cultivate mastery in the performance of fundamental operations with directed numbers	97-99	64-71	59-61	147-148 150	116-118 124-128

Vocabulary	Suggestions
closure	Closure property: Closure under any operation depends on both the particular operation and the universal set of the numbers used. U = the set of all odd integers 3 x 5 = 15, closed under multiplication 3 + 5 = 8, <u>not</u> closed under addition U = the set of positive integers 5 - 8 = ?, not closed under subtraction Use number line to illustrate addition of directed numbers.
addition addend sum term	Use line vectors in adding directed numbers. See: B - pp. 64-71

Have pupils develop and extend this table:

Partial Addition Table for Integers

0	$+1$	$+2$	$+3$	$+4$	$+5$	$+6$	$+7$	$+8$
-1	0	$+1$	$+2$	$+3$	$+4$	$+5$	$+6$	$+7$
-2	-1	0	$+1$	$+2$	$+3$	$+4$	$+5$	$+6$
-3	-2	-1	0	$+1$	$+2$	$+3$	$+4$	$+5$
-4	-3	-2	-1	0	$+1$	$+2$	$+3$	$+4$
-5	-4	-3	-2	-1	0	$+1$	$+2$	$+3$
-6	-5	-4	-3	-2	-1	0	$+1$	$+2$
-7	-6	-5	-4	-3	-2	-1	0	$+1$
-8	-7	-6	-5	-4	-3	-2	-1	0

Note patterns formed.
See: <u>Algebra One</u> -- Hayden and Finan p. 115

Unit I - Sets of Numbers

Topics	Objectives	References A	B	C	D	E
Subtraction		100-104	72-77	62-64	150-153	128-132

Vocabulary	Suggestions

A method for developing the concept of addition of a positive and negative number.

$$(^+7) + (^-3) = [(^+4) + (^+3)] + (^-3)$$
$$= (^+4) + [(^+3) + (^-3)]$$
$$= (^+4) + 0$$
$$= ^+4$$
$$\therefore (^+7) + (^-3) = ^+4$$

$$(^-11) + (^+9) = [(^-2) + (^-9)] + (^+9)$$
$$= (^-2) + [(^-9) + (^+9)]$$
$$= (^-2) + 0$$
$$= ^-2$$
$$\therefore (^-11) + (^+9) = ^-2$$

See: Modern Algebra, First Course
--Johnson, Lendsey, Slesnick pp. 124-126

subtraction
subtrahend
minuend

Subtraction should be developed using one and only one of two methods:

I. The inverse operation of addition

If ----------	then inversely:
$x + y = z$	$z - y = x$
$(^+2) + (^+8) = ^+10$	$(^+10) - (^+8) = ^+2$
$(^-4) + (^+7) = ^+3$	$(^+3) - (^+7) = ^-4$
$(^-3) + (^-2) = ^-5$	$(^-5) - (^-2) = ^-3$
$(^+13) + (^-9) = ^+4$	$(^+4) - (^-9) = ^+13$

II. The additive inverse of the subtrahend

$x - y = x + (^-y)$
$(^+10) - (^+8) = (^+10) + (^-8) = ^+2$
$(^+3) - (^+7) = (^+3) + (^-7) = ^-4$
$(^-5) - (^-2) = (^-5) + (^+2) = ^-3$
$(^+4) - (^-9) = (^+4) + (^+9) = ^+13$

Use number line to explain subtraction of directed numbers.

See: "Modern Mathematics in the Elementary and Junior High Schools" -- Williams, Read, Williams pp. 46-48

Unit I - Sets of Numbers

Topics	Objectives	References A	B	C	D	E
Multiplication		105-107A	78-82	65-67	162-164 165	133-137

Vocabulary	Suggestions
multiplication multiplier multiplicand factor	Have pupils develop and extend this table:

Partial Multiplication Table for Integers

-16	-12	-8	-4	$+4$	$+4$	$+8$	$+12$	$+16$
-12	-9	-6	-3	$+3$	$+3$	$+6$	$+9$	$+12$
-8	-6	-4	-2	$+2$	$+2$	$+4$	$+6$	$+8$
-4	-3	-2	-1	$+1$	$+1$	$+2$	$+3$	$+4$
-4	-3	-2	-1	0	$+1$	$+2$	$+3$	$+4$
$+4$	$+3$	$+2$	$+1$	-1	-1	-2	-3	-4
$+8$	$+6$	$+4$	$+2$	-2	-2	-4	-6	-8
$+12$	$+9$	$+6$	$+3$	-3	-3	-6	-9	-12
$+16$	$+12$	$+8$	$+4$	-4	-4	-8	-12	-16

Note patterns formed.

Unit I - Sets of Numbers

Topics	Objectives	References				
		A	B	C	D	D

Vocabulary	Suggestions

A method of developing the law of signs for multiplication of directed numbers:

Assumptions:

$(2)\ (3) = 6$
$(+2)\ (+3) = +6$
$(+3)\ (0) = 0$
$(-2)\ (0) = 0$

Since:

$(+3)\ (0) = 0$
$(+3)[(+2) + (-2)] = 0$
$(+3)\ (+2) + (+3)\ (-2) = 0$
$(+6) + [(+3)\ (-2)] = 0$

Only -6 added to $+6$ will equal 0.
thus $(+6) + (-6) = 0$
$\therefore\ (+3)\ (-2) = -6$

Since:

$(-2)\ (0) = 0$
$(-2)[(+3) + (-3)] = 0$
$(-2)(+3) + (-2)\ (-3) = 0$
$(-6) + [(-2)\ (-3)] = 0$

Only $+6$ added to -6 will equal 0.
thus $(-6) + (+6) = 0$
$\therefore\ (-2)\ (-3) = +6$

See:D - pp. 162, 163 for another method of developing the law of signs for multiplication of directed numbers.

Unit I - Sets of Numbers

Topics	Objectives	References				
		A	B	C	D	E
Division		109,110	82, 83	68-69	162-164 165	138-141

Vocabulary	Suggestions
division dividend divisor quotient	Division should be developed using one and only one of two methods:

I. The inverse operation of multiplication

If - - - - - - - - -	- then inversely:
$x \cdot y = z$	$z \div y = x$
$(^+2) \cdot (^+5) = {}^+10$	$(^+10) \div (^+5) = {}^+2$
$(^+3) \cdot (^-2) = {}^-6$	$(^-6) \div (^-2) = {}^+3$
$(-\frac{4}{7}) \cdot (^+7) = {}^-4$	$(^-4) \div (^+7) = -\frac{4}{7}$

II. The multiplicative inverse of the divisor

$$x \div y = x \cdot \frac{1}{y} = \frac{x}{y}$$

$$(^+10) \div (^+5) = (^+10) \cdot (+\frac{1}{5}) = \frac{^+10}{+5} = {}^+2$$

$$(^-6) \div (^-2) = (^-6) \cdot (-\frac{1}{2}) = \frac{-6}{-2} = {}^+3$$

$$(^-4) \div (^+7) = (^-4) \cdot (\frac{1}{+7}) = \frac{^-4}{+7} = -\frac{4}{7}$$

Unit II - Variables, Open Expressions, and Open Sentences (7 weeks)

Topics	Objectives	A	B	C	D	E
Variables	To develop the ability to understand the nature of variables	15-16 20	3	1-3,149	1-5	35, 36
Terms and Factors	To develop the ability to recognize the difference between terms and factors	23, 113 33, 46	19-21 122-123 133-134	9-11	18 114-115	40-42 218, 219 276-278

Vocabulary	Suggestions
variable literal notation literal number general number literal coefficient numerical coefficient constant	A variable is a symbol which may represent any of the elements of a specified set. When in a given situation a symbol represents only one number, then the symbol is called a constant. °°°Report on history of algebra See: B - pp. 284-292 C - pp. 238-239 Report on Vieta who introduced use of letters as variables. See: E - p. 62 Report on Hypatia the "first lady" of algebra. See: E - p. 109 Report on Al-Khowarazmi who wrote ilm al-jabr wa'l muqabalah or al-jabr. See: E - pp. 155, 185 Color sound film: Mysterious "X" (30 min.) Association Films 561 Hillgrove Avenue La Grange, Illinois ##"Algebra on the Map" (large chart) D. C. Heath Company 1815 Prairie Avenue Chicago 16, Illinois
term like (similar) terms unlike terms factor literal factor numerical factor base exponent power scientific notation	Distinction should be made between "term" and "factor". An expression written as a product or quotient of numerals or variables or both is called a term. When two or more quantities are multiplied, each of the quantities is called a factor of the product. A coefficient is a factor. An exponent tells how many times another number, called the base, is to be used as a factor. Emphasize the difference between $(^-3)^2$ and $^-3^2$. Note difference in notational meaning of writing numerical symbols adjacent to each other and writing literal symbols adjacent to each other. $34 = 30 + 4$ addition $ab = a \cdot b$ multiplication °°°Introduce zero and negative integers as exponents. Write numbers in bases other than ten using exponential notation. Study logarithms Study history, theory and use of slide rule.

Unit II - Variables, Open Expressions, and Open Sentences

Topics	Objectives	References A	B	C	D	E
	To develop the ability to recognize various types of polynomials					197

Vocabulary	Suggestions
	Sound film: Slide Rule (20 min.) United World 1445 Park Avenue New York 29, New York (Good for after school or Math. Club) ##"How to Choose a Pocket Slide Rule," "Slide Rule? May I Help --," "Elementary Instruction for Operating the Slide Rule" Keuffel and Esser Company Adams and Third Street Hoboken, New Jersey ##"The Slide Rule in a Week" Holt Service Bulletin No. M-4 Holt, Rinehart and Winston, Inc. New York 17, New York ##Manual for Construction and Use of Slide Rule Sample slide rule kit Additional kits (25¢) K. P. Kidd University of Florida Gainesville, Florida
polynomial monomial binomial trinomial	Polynomial is either a monomial or the sum of monomials.
degree	Degree of a variable in a monomial is the number of times the variable occurs as a factor in the monomial. $2x^3y^4z^2$ is of degrees 3 in x, 4 in y, 2 in z. Degree of a monomial (term) is the sum of the degrees in each of its variables. $2x^3y^4z^2$ is of degree 9 because $3 + 4 + 2 = 9$ Degree of a polynomial is the same as greatest of the degrees of its terms. Degrees of the term of $3x^5 + 4x^4y^3 - 2x^2y$ are in order 5, 7, 3 therefore the degree of this polynomial is 7. See: E - p. 197

Unit II - Variables, Open Expressions, and Open Sentences

Topics	Objectives	References A	B	C -	D	E
Fundamental Operations with Variables	To develop the ability to perform fundamental operations with polynomials containing variables					
Addition Subtraction		20-22 100,105 113-118	112-115 136-139	18-20 72-74 85-93	19-22 149,153 155-162 303-304 307-309	198-203
Multiplication Division		22-25 107B-109 111-112 233-237 252-253 260C 261F	86, 87 115-121	15-18 72 74-75 93 114-116 118-119	26-31 164-165 304-306	203-206 215-218
	To develop the ability to remove symbols of inclusion by use of the distributive property	121-123	141-144	11-12 70-71 121-122 128-130	51-53 170-173 312	78, 79
	To develop the ability to group terms with symbols of inclusion				54-57	
Polynomial by Monomial		118-119 237-239	140,141	120	309-311	206-209

Vocabulary	Suggestions

Development of product of powers:

$$x^3 \cdot x^2 = xxx \cdot xx = x^5$$

Development of quotient of powers:

$$\frac{x^9}{x^5} = \frac{x^5 \cdot x^4}{x^5} = \frac{x^5}{x^5} \cdot \frac{x^4}{1} = 1 \cdot \frac{x^4}{1} = x^4$$

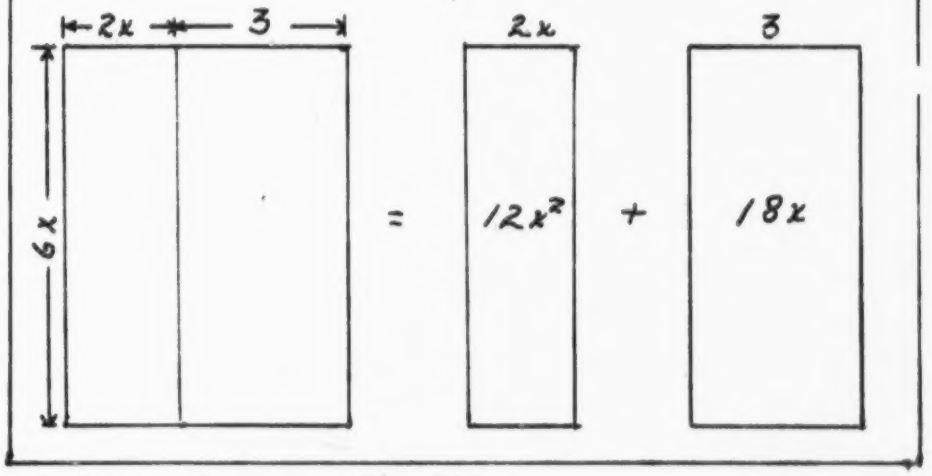
$$\frac{x^5}{x^9} = \frac{x^5}{x^5 \cdot x^4} = \frac{x^5}{x^5} \cdot \frac{1}{x^4} = 1 \cdot \frac{1}{x^4} = \frac{1}{x^4}$$

Emphasis should be placed on use of the distributive property in removing symbols of inclusion.

Some literal polynomials should be used in drill exercises.

Multiplying a polynomial by a monomial:

$$6x(2x + 3) = 6x(2x) + 6x(3) = 12x^2 + 18x$$



See: E - pp. 206, 207

Unit II - Variables, Open Expressions, and Open Sentences

Topics	Objectives	References A	B	C	D	E
		119-120 253-256	148-149	124,125	309-311	219-221
Polynomial by Polynomial		244-246	144-146	123	313-320	209-210 213, 214
		256-258	150-153	4-5 126-127	324-328	221-223

Vocabulary	Suggestions

Dividing a polynomial by a monomial:
Stress use of distributive property to prevent $\frac{25 + 5}{5} = 25 + 1$

Use:
$$\frac{25 + 5}{5} = \frac{1}{5}(25 + 5)$$
$$= \frac{1}{5}(25) + \frac{1}{5}(5)$$
$$= 5 + 1$$
$$= 6$$
$$\frac{bx + by}{b} = \frac{1}{b}(bx + by)$$
$$= \frac{1}{b}(bx) + \frac{1}{b}(by)$$
$$= 1x + 1y$$
$$= x + y$$

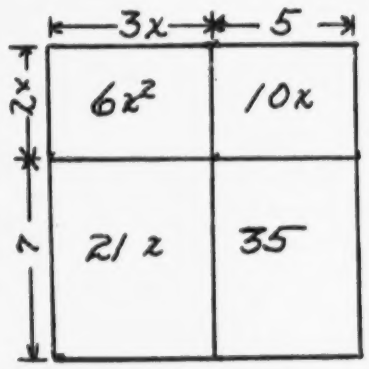
Multiplying a polynomial by a polynomial:

$(3x + 5)(2x + 7) = (3x)(2x + 7) + (5)(2x + 7)$

$(3x + 5)(2x + 7) = (3x)(2x) + (3x)(7) + (5)(2x) + (5)(7)$

$(3x + 5)(2x + 7) = 6x^2 + 21x + 10x + 35$

$(3x + 5)(2x + 7) = 6x^2 + 31x + 35$



See: D - pp. 313,314

Dividing a polynomial by a polynomial:
See: D - pp. 324-326

Unit II - Variables, Open Expressions, and Open Sentences — References

Topics	Objectives	A	B	C	D	E
Open Expressions	To develop the ability to translate verbal expressions into open (algebraic) expressions	37, 38 126	6-7, 9 168	5, 6 13-14 96-98	7-11 76-79	42 51-55
	To develop the ability to translate open expressions into verbal expressions	17		4,5,14 24		49-51
	To develop the ability to determine the numerical value of an algebraic expression	17-19 34	8,21-22 87-88	4,5-6 76,26	13-14 33 88-90 165-168	36-39 43, 44
Symbols of Equality and Inequality	To develop the ability to use symbols of equality and inequality	16,17 77,78 261-264	92-93		15-18 74-75	5-10 114,115

Vocabulary	Suggestions
expression algebraic open	Letters of the alphabet, such as x, are generally used instead of the blank in an open expression and thus act as a "spaceholder" or "placeholder". Examples: $\frac{\quad}{7 - \quad} + 5$ or $x + 5$ or $7 - x$ The symbol "x" is called a variable and the set of possible numerical replacements for "x" is called the replacement set. When discussing a variable, the universal set should always be specified. If U = the set of all natural numbers, then in the expression 3x, the replacement set for x is N = {1, 2, 3, ...}. (3 · 5) shows a particular replacement of x from its replacement set and 15 is the value of the expression when x is replaced by 5. Emphasize the need for stating exactly what <u>number</u> the variable represents. x represents the number of feet in the width; not x represents the width. °°°Cryptography See E - p. 33 and "Secret Codes, Remainder Arithmetic, and Matrices" by Peck
evaluate replacement set	
"is equal to" = "is not equal to" ≠ "is greater than" > "is not greater than" ≯ "is less than" < "is not less than" ≮ "is greater than or equal to" ≥ "is less than or equal to" ≤	°°°Report on Recorde and Harriot who used and promoted use of symbols =, >, <.

Topics	Objectives	References				
		A	B	C	D	E
						69-70
						159-160

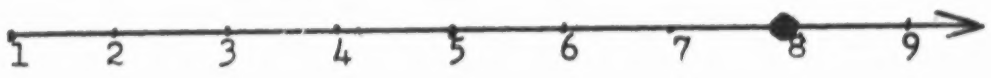
Vocabulary	Suggestions
reflexive property	Introduce: Reflexive property of equality: $a = a$ (any number is equal to itself)
symmetric property	Symmetric property of equality: If $a = b$, then $b = a$ (an equality is reversible)
transitive property	Transitive property of equality: If $a = b$, and $b = c$, then $a = c$ (two numbers are equal to each other if each of them equals a third number)
order property	Order property of numbers: For each pair of directed numbers, a and b, one and only one of the following statements is true: $a < b$, $a = b$, $a > b$.
	Transitive property of inequality: If $a < b$ and $b < c$, then $a < c$ If $a > b$ and $b > c$, then $a > c$

Unit II - Variables, Open Expressions, and Open Sentences

Topics	Objectives	References A	B	C	D	E
Open Sentences	To develop the ability to translate verbal statements into open (algebraic) sentences	53,54A 126	1, 89 90 24-31	7, 44 25	118-123	44, 45 115,116
Set-builder	To develop the ability to use the set-builder symbol		200-201			
	To develop the ability to translate open sentences into verbal sentences				107-109	
	To develop the ability to form, intuitively, true closed sentences from open sentences	34B-35 38 49-51 54B 57-58 60 78-79	10 94-96	26	22-26 49-50	57, 58 46-49 118,119

Vocabulary	Suggestions
sentence algebraic open formula	An open (algebraic) sentence is a statement composed of open (algebraic) expressions related by symbols such as: $=$, $\neq$, $>$, $<$, $\leq$, or $\geq$. These symbols of relationship are equivalent to the verbs in a sentence.
set-builder $\{x \mid \}$	°°°Introduce the symbol $\{x \mid \}$ referred to as the "set-builder". $\{x \mid x + 9 = 17\}$ is read "the set of all x such that $x + 9 = 17$. $\{x \mid x + 9 = 17\} = \{8\}$
equation inequation	Open sentences are equations or inequations. Any sentence using the symbol $=$ is called an equation. Any sentence using a symbol of inequality is an inequation.
evaluate closed sentence	When values are substituted for variables in an equation or inequation either true or false statements are obtained. U = the set of all natural numbers. An open sentence such as $x + 9 = 17$ divides U into two sets: A = all replacements for x that make $x + 9 = 17$ false.

B = all replacements for x that make $x + 9 = 17$ true. In this case B is the solution set.
Note that set B contains just one element 8.

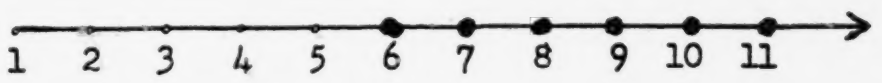
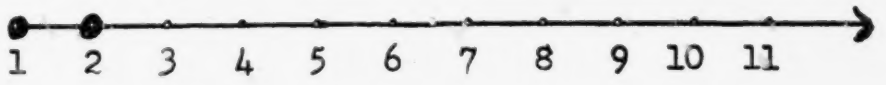
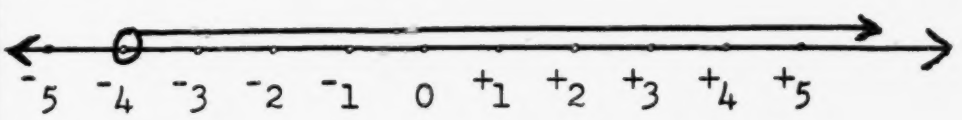


°°°Report on Ahmes Papyrus.
See: E - p. 224
Report on Diophantus. (Diophantine equations are equations whose replacement sets are restricted to subsets of integers.)
See: E pp. 393-394

##"Hydraulic Formulas
New York Air Brake Company
Watertown Division
Starbuck Avenue
Watertown, New York

Unit III - Equations and Inequations (5 weeks)

Topics	Objectives	References A	B	C	D	E
Graphing Equations and Inequations in One Variable	To develop the ability to graph on a number line the solution sets of equations and inequations in one variable		204-208			16-17

Vocabulary	Suggestions
	U = the set of all natural numbers. $\{x \mid x \geq 6\} = \{6, 7, 8, \ldots\}$  $\{p \mid p + 3 \leq 5\} = \{2, 1\}$  U = the set of all real numbers. $\{y \mid y\ ^{-}4\}$ = {all real numbers greater than $^{-}4$}  See: "Modern Mathematics in the Elementary and Junior High Schools" -- Williams, Read, Williams "Modern Mathematics: Topics and Problems"-- Aiken and Beseman "Worktext in Modern Mathematics" -- Marston "Supplementary Units in Contemporary Arithmetic and Elementary Algebra" -- Stein
locus	°°°Introduce concept of locus at this time. A locus is the set of points, and only those points, that satisfy a given condition. The solution set of the equation or inequation corresponds to the set of points that make up the locus. Thus the graph pictures the locus of the equation or inequation or the solution set. See: "Report of the Commission on Mathematics Appendices" -- CEEB

Unit III - Equations and Inequations

Topics	Objectives	References A	B	C	D	E
Function	To develop the concept of function			150,151	129-130	438-439
Dependence	To develop the ability to understand the dependence of one variable upon another	25-33 183-185 347-348	31-32	147-149	126-129 219-220	
Relationship	To develop the ability to show the relationship between: 1. verbal statements and open sentences 2. tables and graphs 3. tables and open sentences 4. graphs and open sentences	30, 39 40		152-156		

Vocabulary	Suggestions
function ordered pair	A function is a set of ordered pairs of numbers. In the function $A = s^2$, one number pair is (4, 16). The 16 is the value of the function for $s = 4$. If $s = 7$, the number pair in $A = s^2$ is (7, 49) and the value of the function is 49.
dependence independent variable	The first variable in the pair is call the <u>independent variable</u>.
dependent variable	The second variable in the pair is the dependent <u>variable</u>.
	In the formula $C = \pi d$, the ordered pair (d, C), d is the independent variable and C is the dependent variable.
	The equation $y = 3x + 7$ describes a function. The set of all (x,y) that makes this become a true statement is the function. It is possible to
table	list some of (x, y) in a table.

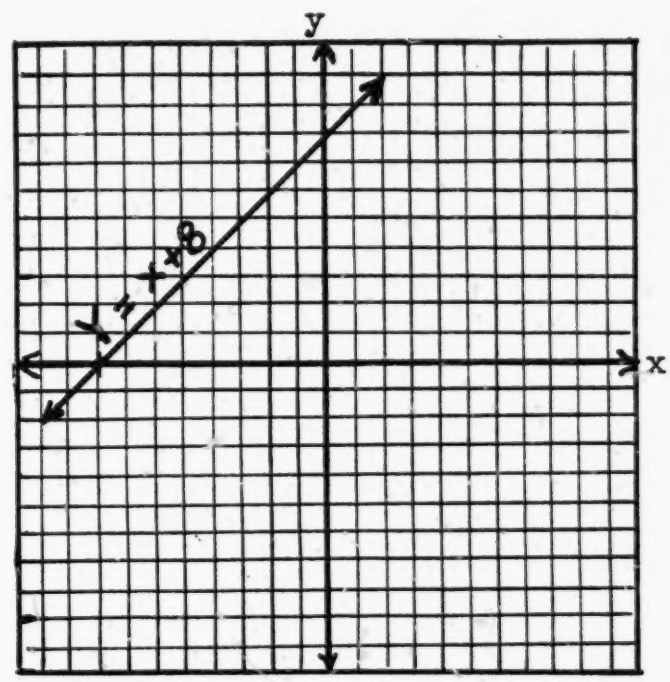
If x =	1	2	3	4	5
then y =	10	13	16	19	22

relationship

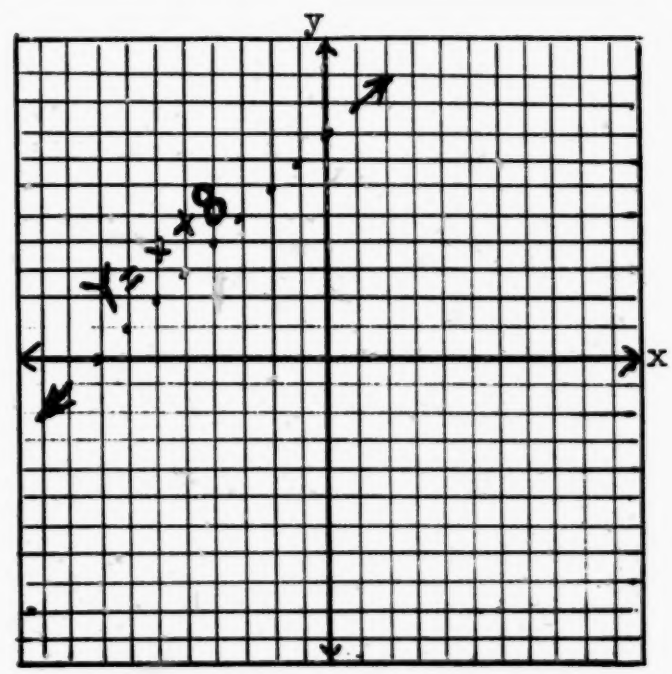
Unit III - Equations and Inequations

Topics	Objectives	References A	B	C	D	E
Graphing Equations and Inequations in Two Variables	To develop the ability to graph on a plane the solution sets of equations and inequations in two variables	164-167 180-181 224D 225F 226G	225-226	182-190	130-134 220-222 260-261	333-337 340-342

Vocabulary	Suggestions
plane x-axis y-axis x-intercept y-intercept	○○○Report on René Descartes who invented coordinate geometry. See: E - p. 365 Open sentences in two variables: U = set of all real numbers $y = x + 8$ $\{(x,y) \mid y = x + 8\}$ is read: "the set of ordered pairs (x,y) such that $y = x + 8$



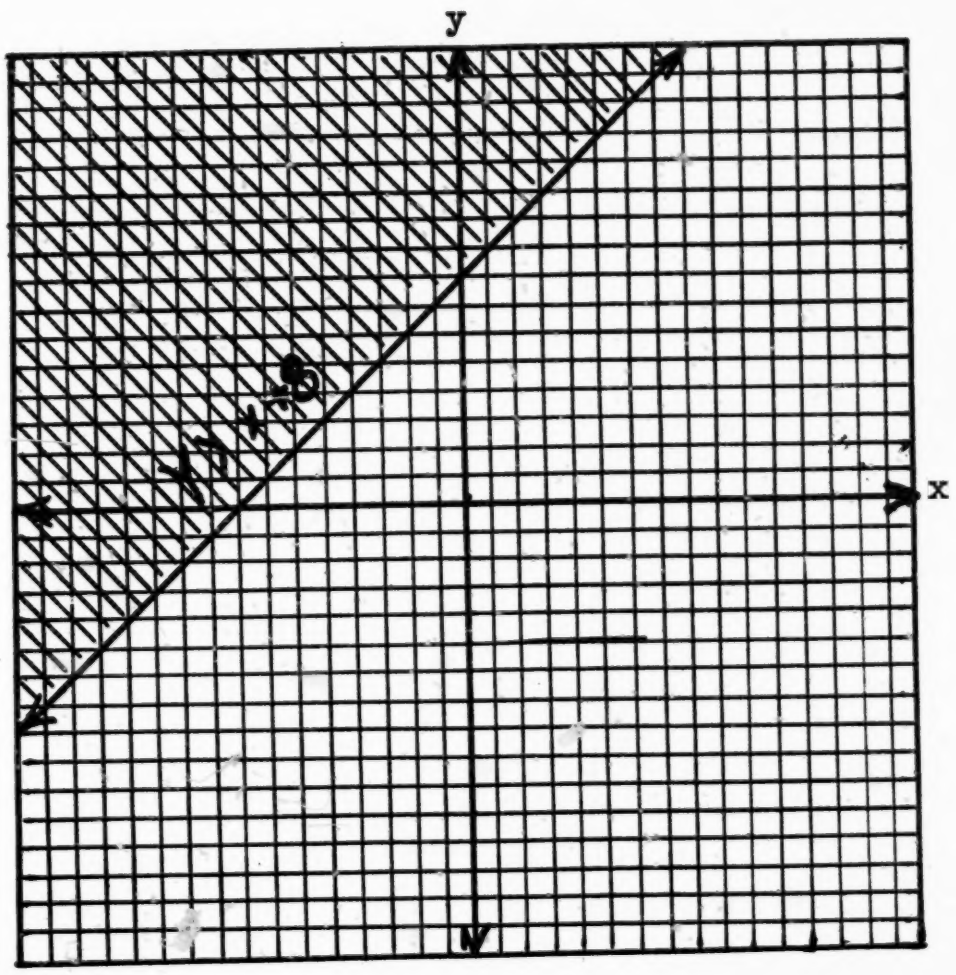
U = the set of all natural numbers
$\{(x,y) \mid y = x + 8\}$



Unit III - Equations and Inequations

Topics	Objectives	References A	B	C	D	E
						350-353 356G-J

Vocabulary	Suggestions
	ooo Graphing pictures using inequalities See: "The Mathematics Teacher" -- April, 1960 pp. 295-296
	U = set of all real numbers $\{(x,y) \mid y > x + 8\}$
half-plane	Note: Graph is the "half-plane" above the line representing $y = x + 8$



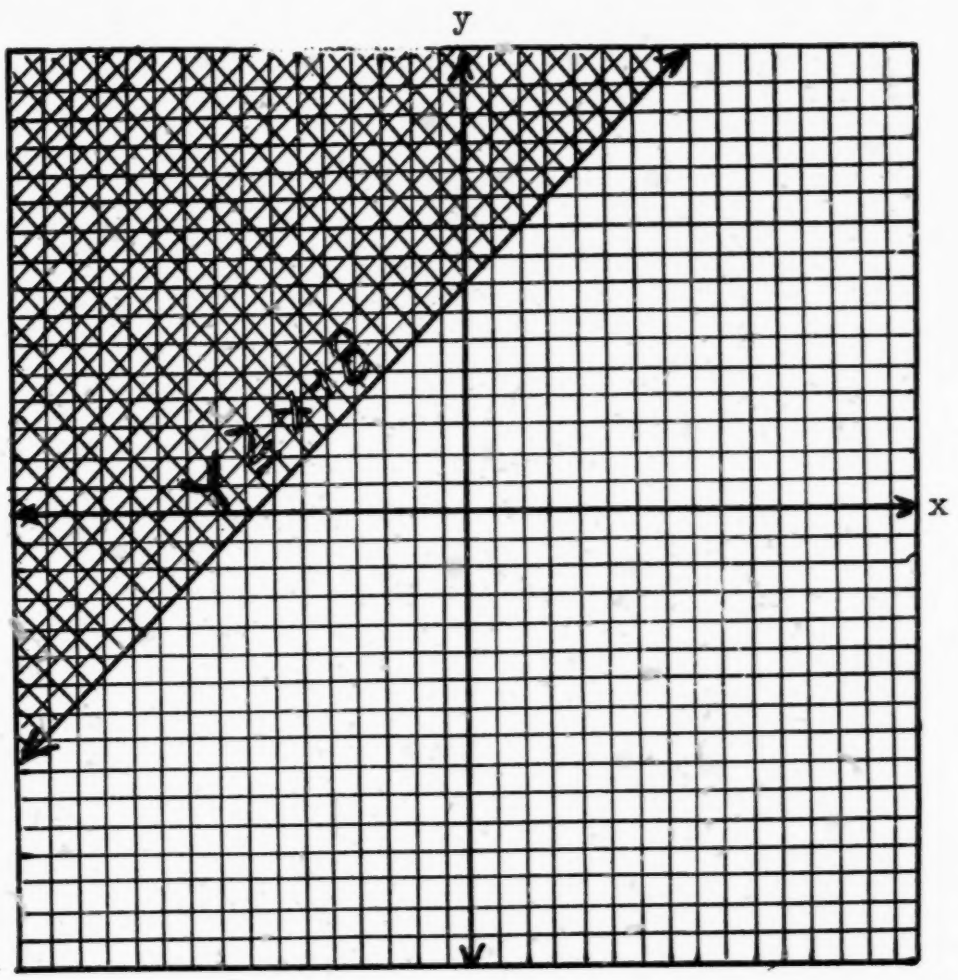
Unit III - Equations and Inequations

Topics	Objectives	References				
		A	B	C	D	E
Properties of Equality and Inequality	To develop the ability to understand the properties of equality and inequality		184			80, 83

Vocabulary	Suggestions

U = set of all real numbers

$\{(x,y) \mid y \geq x + 8\}$



The solution set of $y \geq x + 8$ is the union of the two solution sets $y > x + 8$ and $y = x + 8$. Its graph is the "half-plane" representing the solution set of $y > x + 8$, and the line which represents the solution set of $y = x + 8$.

See: "Report of the Commission on Mathematics Appendices" -- CEEB

"Adventures in Graphing" -- Glenn and Johnson

equivalent equations

Equivalent equations (equations having the same solution set) may be developed based on these properties:

Addition property of equality
If $a = b$, then $a + c = b + c$

Subtraction property of equality
If $a = b$, then $a - c = b - c$

Multiplication property of equality
If $a = b$, then $a \cdot c = b \cdot c$

Division property of equality
If $a = b$, and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$

Unit III - Equations and Inequations

Topics	Objectives	References A	B	C	D	E
Solution Sets of Equations and Inequations						
I. Deriving Solution Sets of Equations Using Inverse Operations	To develop the ability to derive the solution sets of equations by use of inverse operations	51-52 55-56 58, 59 61-65 75-77 124-125 239-240 247-248 260E	166-168 171-172 173-174 175 176-177 180-182 183-184 146-147 201-204	28-42 77-78 130-131	57-70 173-174 321,322	81-83 84-86 88, 89 91-94 157-159

Vocabulary	Suggestions
equivalent inequations	Equivalent inequations (inequations having the same solution set) may be developed based on these properties: Addition property of inequality If $a < b$, then $a + c < b + c$ If $a > b$, then $a + c > b + c$ Subtraction property of inequality If $a < b$, then $a - c < b - c$ If $a > b$, then $a - c > b - c$ Multiplication property of inequality If $a < b$, then $ac < bc$, when $c > 0$ $ac = bc$, when $c = 0$ $ac > bc$, when $c < 0$ Division property of inequality If $a < b$, then $\frac{a}{c} < \frac{b}{c}$, when $c > 0$ $\frac{a}{c} > \frac{b}{c}$, when $c < 0$ Note: Use <u>either</u> I (the method of inverse operations) <u>or</u> II (the method of additive and multiplicative inverses) in deriving solution sets of equations and inequations. I. Deriving solution sets of equations by use of inverse operations: $x - 5 = 12$ $x - 5 + 5 = 12 + 5$ $x = 17$ $x + 8 = 15$ $x + 8 - 8 = 15 - 8$ $x = 7$ $\frac{x}{3} = 8$ $3 \cdot \frac{x}{3} = 3 \cdot 8$ $x = 24$ $5x = 30$ $\frac{5x}{5} = \frac{30}{5}$ $x = 6$ See: E - pp. 80-84

Unit III - Equations and Inequations

Topics	Objectives	References A	B	C	D	E
I. Deriving Solution Sets of Inequations Using Inverse Operations	To develop the ability to derive the solution sets of inequations by use of inverse operations	130-133 80-83	163-164 184-187 99-100			159-163

Vocabulary	Suggestions

I. Deriving solution sets of inequations by use of inverse operations:

$$x - 5 > 12$$
$$x - 5 + 5 > 12 + 5$$
$$x > 17$$

$$x + 8 < 15$$
$$x + 8 - 8 < 15 - 8$$
$$x < 7$$

$$\frac{x}{3} > 8$$
$$3 \cdot \frac{x}{3} > 3 \cdot 8$$
$$x > 24$$

$$-\frac{x}{7} > 9$$
$$(^-7)\ (-\frac{x}{7}) < (^-7)\ (9)\ ^{\circ}$$
$$x < ^-63$$

$$5x < 30$$
$$\frac{5x}{5} < \frac{30}{5}$$
$$x < 6$$

$$^-4x < 32$$
$$\frac{^-4x}{^-4} > \frac{32}{^-4}\ ^{\circ}$$
$$x > ^-8$$

°Note change of inequality symbol.

See: E - pp. 159-162

Unit III - Equations and Inequations

Topics	Objectives	References A	B	C	D	E
II. Deriving Solution Sets of Equations Using Additive and Multiplicative Inverses	To develop the ability to derive the solution sets of equations by means of additive and multiplicative inverses					

Vocabulary	Suggestions

II. Deriving solution sets of equations by means of additive and multiplicative inverses:
The solution set of an equation involving the four fundamental operations may be derived by using only the operations of addition and multiplication.
(Use the additive inverse or the multiplicative inverse).

$$
\begin{aligned}
a - 5 &= 17 \\
(^+a) + (^-5) &= (^+17) \\
(^+a) + (-5) + (^+5) &= (^+17) + (^+5) \\
^+a &= ^+22 \\
a &= 22
\end{aligned}
$$

$$
\begin{aligned}
a + 6 &= 13 \\
(^+a) + (^+6) &= (^+13) \\
(^+a) + (^+6) + (^-6) &= (^+13) + (^-6) \\
^+a &= ^+7 \\
a &= 7
\end{aligned}
$$

$$
\begin{aligned}
\frac{a}{7} &= 3 \\
\frac{^+a}{^+7} &= ^+3 \\
\left(\frac{1}{^+7}\right)\cdot\frac{(^+a)}{1} &= \frac{^+3}{1} \\
\left(\frac{^+7}{1}\right)\cdot\left(\frac{1}{^+7}\right)\cdot\left(\frac{^+a}{1}\right) &= \left(\frac{^+7}{1}\right)\cdot\left(\frac{^+3}{1}\right) \\
^+a &= ^+21 \\
a &= 21
\end{aligned}
$$

$$
\begin{aligned}
9a &= 27 \\
\frac{(^+9)}{1}\cdot\frac{(^+a)}{1} &= \frac{(^+27)}{1} \\
\left(\frac{1}{^+9}\right)\cdot\left(\frac{^+9}{1}\right)\cdot\left(\frac{^+a}{1}\right) &= \left(\frac{1}{^+9}\right)\cdot\left(\frac{^+27}{1}\right) \\
^+a &= ^+3 \\
a &= 3
\end{aligned}
$$

Unit III - Equations and Inequations

Topics	Objectives	References A	B	C	D	E
II. Deriving Solution Sets of Inequations Using Additive and Multiplicative Inverses	To develop the ability to derive the solution sets of inequations by means of additive and multiplicative inverses					

Vocabulary	Suggestions

II. Deriving solution sets of inequations by means of additive and multiplicative inverses:

$$\begin{aligned} a - 5 &> 17 \\ (+a) + (-5) &> (+17) \\ (+a) + (-5) + (+5) &> (+17) + (+5) \\ (+a) &> (+22) \\ a &> 22 \end{aligned}$$

$$\begin{aligned} a + 6 &< 13 \\ (+a) + (+6) &< (+13) \\ (+a) + (+6) + (-6) &< (+13) + (-6) \\ (+a) &< (+7) \\ a &< 7 \end{aligned}$$

$$\begin{aligned} \frac{a}{7} &> 3 \\ \frac{(+a)}{(+7)} &> \frac{(+3)}{(+1)} \\ \frac{(+1)}{(+7)}\ \frac{(+a)}{(+1)} &> \frac{(+3)}{(+1)} \\ \frac{(+7)}{(+1)}\ \frac{(+1)}{(+7)}\ \frac{(+a)}{(+1)} &> \frac{(+7)}{(+1)}\ \frac{(+3)}{(+1)} \\ \frac{(+a)}{(+1)} &> \frac{(+21)}{(+1)} \\ a &> 21 \end{aligned}$$

$$\begin{aligned} \frac{b}{-6} &< \frac{4}{1} \\ \frac{(+b)}{(-6)} &< \frac{(+4)}{(+1)} \\ \frac{(+1)}{(-6)}\ \frac{(+b)}{(+1)} &< \frac{(+4)}{(+1)} \\ \frac{(-6)}{(+1)}\ \frac{(+1)}{(-6)}\ \frac{(+b)}{(+1)} &> \frac{(-6)}{(+1)}\ \frac{(+4)}{(+1)}\ ^{\circ} \\ \frac{(+b)}{(+1)} &> \frac{(-24)}{(+1)} \\ b &> -24 \end{aligned}$$

$$\begin{aligned} 9a &> 27 \\ \frac{(+9)}{(+1)}\ \frac{(+a)}{(+1)} &> \frac{(+27)}{(+1)} \\ \frac{(+1)}{(+9)}\ \frac{(+9)}{(+1)}\ \frac{(+a)}{(+1)} &> \frac{(+1)}{(+9)}\ \frac{(+27)}{(+1)} \\ \frac{(+a)}{(+1)} &> \frac{(+3)}{(+1)} \\ a &> 3 \end{aligned}$$

$$\begin{aligned} -8a &> 24 \\ \frac{(-8)}{(+1)}\ \frac{(+a)}{(+1)} &> \frac{(+24)}{(+1)} \\ \frac{(+1)}{(-8)}\ \frac{(-8)}{(+1)}\ \frac{(+a)}{(+1)} &< \frac{(+1)}{(-8)}\ \frac{(+24)}{(+1)}\ ^{\circ} \\ \frac{(+a)}{(+1)} &< \frac{(-3)}{(+1)} \\ a &< -3 \end{aligned}$$

°Note change of inequality symbol.

Unit III - Equations and Inequations

Topics	Objectives	References A	B	C	D	E
Deriving Solution Sets of Literal Equations and Inequations	To develop the ability to determine the solution sets of literal equations and inequations	72-74	192-193		110-111 140-141	
Deriving Solution Sets for Verbal Problems (One Variable)	To develop the ability to solve verbal problems using one variable	66-71 127-130	170 172-173 174 175-176 177-179 182 188-191	42-43 45-46 98-106	79-87 93-94 112-113 116-118 123-126 175-181	56-57 58-60 87 89-91 94-95 119,120 166-184 211-213 214,215

Vocabulary	Suggestions
literal equation literal inequation	Suggested method for finding the solution set for a verbal problem: 1. Choose a variable to represent the <u>number</u> needed to satisfy the conditions of the problem. 2. Translate the conditions of the problem into an open sentence. 3. Determine the solution set of the open sentence. 4. Check to determine whether the solution set satisfies the given conditions.

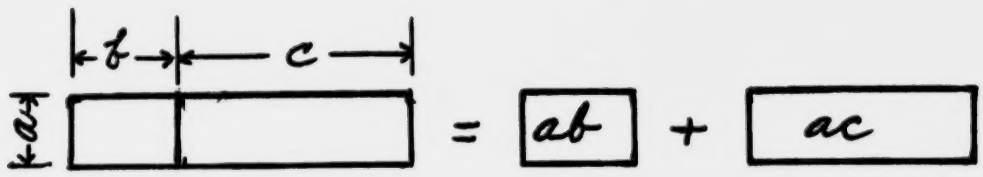
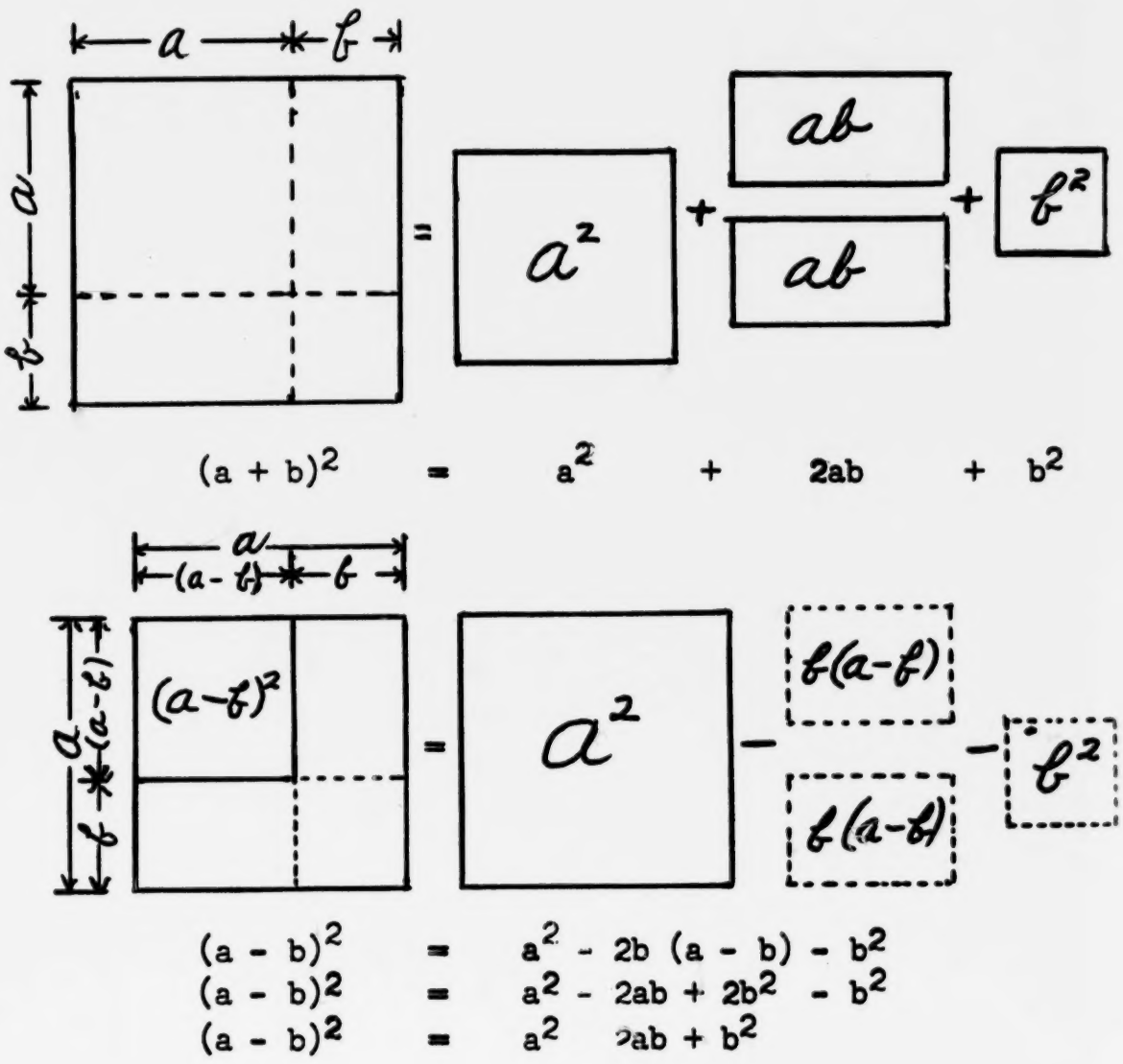
Unit IV - Systems of Equations and Inequations (3 weeks)

Topics	Objectives	References A	B	C	D	E
Deriving Solution Sets of Systems of Equations and Inequations						
Graphing	To develop the ability to determine the solution sets of systems of equations and inequations by graphing	168-179 182-183 202-204 224-225E	208-210 225-253 260-262	191-198 229-230	223 260-262 332-334	343-350 367-370 379-381
Addition or Subtraction	To develop the ability to determine the solution sets of systems of equations by using addition or subtraction	195-199	256-260 263-264	215-218	227-231	370-372 374-377
Substitution	To develop the ability to determine the solution sets of systems of equations by using substitution	200-201	254-256 265	219-221	224-227	378
Deriving Solution Sets for Verbal Problems (Two Variables)	To develop the ability to solve verbal problems using two variables	204-224 241-243 248-251 258-260	265-273	222-229	233-235	372-374 377, 379 381-386

Vocabulary	Suggestions
simultaneous equations simultaneous inequations	The solution set of two or more equations and/or inequations is the <u>intersection</u> set.
inconsistent parallel dependent coincide independent intersect	Refer to previous sections on graphing solution sets of equations and inequations. Sound film: <u>Graphing Linear Equations</u> (11 min.) Coronet Films 65 E. South Water Street Chicago 1, Illinois
slope intercept	Introduce slope-intercept form of equation: $y = mx + b$ °°°Find the equation of a straight line when the slope and y-intercept are known. See: C - pp. 197-198 B - pp. 260-262 Solve simultaneous equations using second order determinants Linear Programing See: E - pp. 461-462

Unit V - Products and Factors (4 weeks)

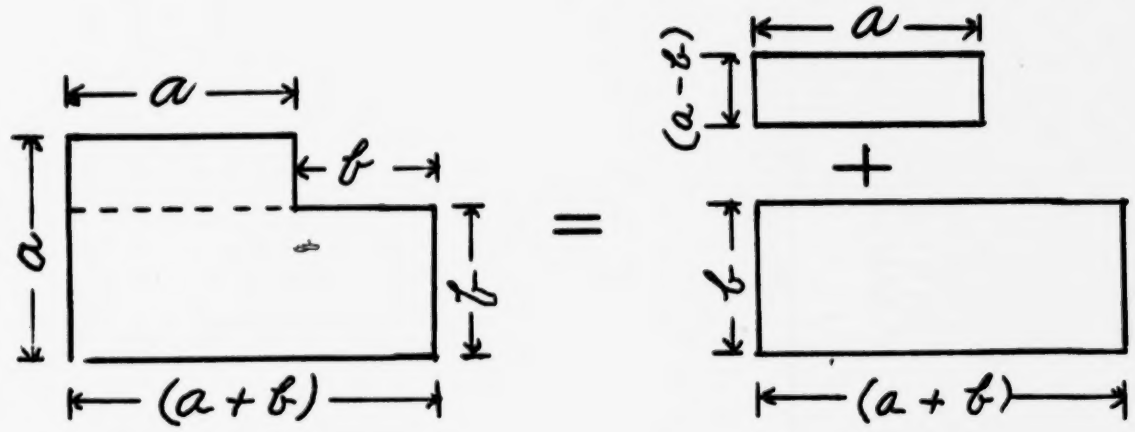
Topics	Objectives	References A	B	C	D	E
Distributive Property and Its Inverse	To develop the ability to use the distributive property and its inverse				335-336	237-240
Prime Factors	To develop the ability to factor a polynomial expression into its prime components	274	295-296	240-242	338	
Common Monomial Factors	To develop the ability to discover the largest common monomial factor in a polynomial	273-274 275-276	297-298	243-246	336-337 339-341	241-244
Binomial Factors	To develop the ability to discover the binomial factors of a polynomial	276-301 305-307	299-314	247-263	342-362	245-264

Vocabulary	Suggestions
prime factor	Emphasis should be placed on deriving prime factors.
common monomial factor largest common monomial factor	 $a(b + c) = ab + ac$
binomial factor	See: E - pp. 248-251  $(a + b)^2 = a^2 + 2ab + b^2$ $(a - b)^2 = a^2 - 2b(a - b) - b^2$ $(a - b)^2 = a^2 - 2ab + 2b^2 - b^2$ $(a - b)^2 = a^2 - 2ab + b^2$

Unit V - Products and Factors

Topics	Objectives	References				
		A	B	C	D	E

Vocabulary	Suggestions

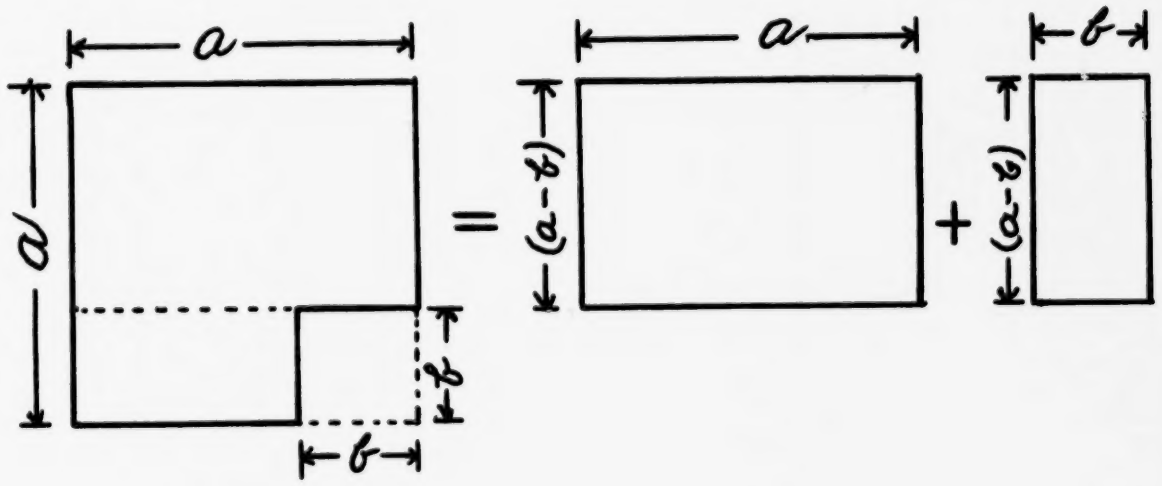


$a^2 + b^2 = b(a + b) + a(a - b)$

$a^2 + b^2 = ab + b^2 + a^2 - ab$

$a^2 + b^2 = b^2 + a^2$

Note: Although $a^2 + b^2$ is non-factorable in the real number system, it is factorable in the complex number system.



$a^2 - b^2 = a(a - b) + b(a - b)$

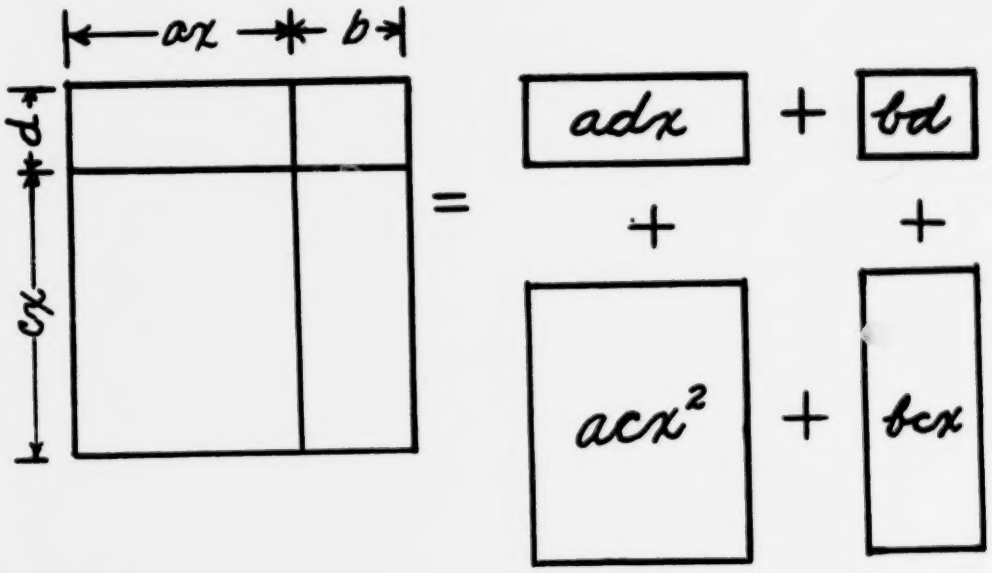
$a^2 - b^2 = (a - b)(a + b)$

Unit V - Products and Factors

Topics	Objectives	References				
		A	B	C	D	E

Vocabulary	Suggestions

One method of factoring a quadratic trinomial:



$(ax + b)(cx + d) = acx^2 + (adx + bcx) + bd$

$(ax + b)(cx + d) = acx^2 + (ad + bc)\,x + bd$

$(ax + b)(cx + d) = acx^2 + adx + bcx + bd$

Example:

$6m^2 + 17m - 14$

$6 \cdot (-14) = -84$

$(+21)(-4)$

$6m^2 + 17m - 14$

$(+21) + (-4)$

$acx^2 + (ad + bc)\,x + bd \longleftrightarrow 6m^2 + (21 - 4)\,m - 14$

$acx^2 + adx + bcx + bd \longleftrightarrow 6m^2 + 21m - 4m - 14$

$ax(cx + d) + b(cx + d) \longleftrightarrow 3m(2m+7) - 2(2m+7)$

$(ax + b)(cx + d) \longleftrightarrow (3m - 2)(2m + 7)$

Use geometric methods to develop algebraic identities.
See: E - pp. 243-244

°°°Use geometric methods to develop more complicated algebraic identities.
Factor sum or difference between two cubes.
See: C - p. 262

Unit V - Products and Factors

Topics	Objectives	References A	B	C	D	E
Deriving Solution Sets of Quadratic Equations by Factoring	To develop the ability to derive the solution sets of quadratic equations by factoring	301-305	315-318	264-266	367-378	264-270

Vocabulary	Suggestions
quadratic equation standard form	Standard form of a quadratic equation: $ax^2 + bx + c = 0$ Emphasize that setting factors of a member of a quadratic equation equal to zero is dependent upon having the original quadratic equation in standard form. No principle applies <u>directly</u> to an equation such as: $(2x + 1)(3x - 7) = 3$ This type of equation should be changed to standard form: $6x^2 + 3x - 14x - 7 = 3$ $6x^2 - 11x - 10 = 0$ Emphasize that the solution set (roots) will have the same number of members as the degree of the equation. $x^3 - 49x = 0$ $x(x^2 - 49) = 0$ $x(x - 7)(x + 7) = 0$ $x = 0 \quad x = +7 \quad x = -7$ Caution: A root may be lost by dividing both members of an equation by the variable. Also extraneous roots may be introduced by multiplying both members of an equation by the variable. Members of the solution set (roots) of an equation are the values of the variable which satisfy the original equation.

Unit VI - Algebraic Fractions (5 weeks)

Topics	Objectives	References A	B	C	D	E
Arithmetic Fractions	To re-inforce the understanding of arithmetic fractions and operations with them	317-319 336 337-338			36-44	
Algebraic Fractions	To develop the meaning of an algebraic fraction as an indicated division		325-326	273-275		281-282
Transforming Fractions	To develop the ability to transform:					
	1. fractions to mixed expressions	322-324				
	2. mixed expressions to fractions	337				302-304
	3. fractions to lower terms	319 -322	328-330	276-281	189-190	283-285
	4. fractions to higher terms		327-328	276,277		
	5. fractions by changing of signs	334-335	341-343	295-297	398-401	

Vocabulary	Suggestions
term numerator denominator	Show parallelism between arithmetic and algebraic fractions. Use work with arithmetic fractions prior to similar work in algebraic fractions. Discuss members of replacement set which must be excluded in certain fractions. $\frac{1}{x-3}$ $x \neq 3$ $\frac{2x}{x+4}$ $x \neq -4$ $\frac{6}{x^2-9}$ $x \neq -3$, $x \neq +3$ See: E - p. 281 Analyze fractions using the distributive property: $\frac{x+y}{z} = \frac{1}{z}(x+y) = \frac{1}{z}(x) + \frac{1}{z}(y)$
	Writing the terms of a fraction in prime factored form is helpful in reducing fractions to lowest terms. Avoid use of term "cancellation". Instead use "divide numerator and denominator by a common factor".

Unit VI - Algebraic Fractions

Topics	Objectives	References				
		A	B	C	D	E
Fundamental Operations	To develop the ability to perform fundamental operations with fractions				44-45	
Multiplication Division		324-326 327-329	330-333 334-335	116-117 281-283 117-118 284-286	402-405 417,418	292-294 295-297
Addition Subtraction		329-334 349-351	335-341 343	94-96 288-294	191-195 405-408	297-302
Complex Fractions	To develop the ability to simplify complex fractions	338-339	343-345	287-288	394-397 417,418	304-305
Deriving solution sets of equations and inequations involving fractions	To develop the ability to derive solution sets of equations and inequations involving fractions	339-347	350-368	132-135 307-327	195-210 233-251 408-416 419-420	306-318 386-387

Vocabulary	Suggestions
least common denominator	Use prime factors in determining least common denominator: $\frac{1}{4a} + \frac{1}{12a^2} + \frac{1}{8a^3} = \frac{1}{2\cdot 2\cdot a} + \frac{1}{2\cdot 2\cdot 3\cdot a\cdot a} + \frac{1}{2\cdot 2\cdot 2\cdot a\cdot a\cdot a}$ LCD is $2\cdot 2\cdot 2\cdot 3\cdot a\cdot a\cdot a = 24a^3$
complex fraction	Simplifying complex fractions:

Method I

$$\frac{\frac{1}{7}}{\frac{4}{7}} \times \frac{7}{7} = \frac{1}{4}$$

$$\frac{1\frac{3}{5}}{2\frac{2}{3}} = \frac{\frac{8}{5}}{\frac{8}{3}} \times \frac{15}{15} = \frac{24}{40} = \frac{3}{5}$$

Method II

$$\frac{\frac{1}{7}}{\frac{4}{7}} \times \frac{\frac{7}{4}}{\frac{7}{4}} = \frac{\frac{1}{4}}{1} = \frac{1}{4}$$

$$\frac{1\frac{3}{5}}{2\frac{2}{3}} = \frac{\frac{8}{5}}{\frac{8}{3}} \times \frac{\frac{3}{8}}{\frac{3}{8}} = \frac{\frac{3}{5}}{1} = \frac{3}{5}$$

Note: Not all equations containing fractions are fractional equations. In a <u>fractional equation</u> the letter representing the unknown number appears in a denominator.

Examples: The equation $\frac{a}{3} - \frac{a}{6} = 1$ is an equation containing fractions, but it is not a fractional equation.

The equation $\frac{2a - 3}{a} + 1 = 2$ is a fractional equation.

Unit VII - Ratio, Proportion, and Variation (2 weeks)

Topics	Objectives	References A	B	C	D	E
Ratios	To develop an understanding of ratio as: 1. a method of comparing like quantities by division 2. an ordered pair of numbers	363-370	374-376 380-381	161-164	95-101	286-291
Proportion	To develop the ability to understand a proportion as two equal ratios	371-387	376-379 382-386			443-444 445-447
	To develop the ability to set up a proportion				105-106	
	To develop the ability to derive the solution set of an open proportion					
Variation	To develop an understanding of variation	388-392	386-394	165-167	263-284	442 444-445 447-456
	To recognize direct and inverse variation in an equation					
	To find the solution set for formulas which express direct and inverse variation					

Vocabulary	Suggestions
ratio continued ratio abstract number	A ratio of one number to another is the quotient of the first number divided by the second. A ratio is always an abstract number.
ordered pair	°°°Study of musical ratios. The eight-note scale is attributed to Pythagoras. See: Algebra, Book One by Welchons, Krickenberger, Pearson -- pp. 501-502
proportion means extremes scale drawing similar triangles corresponding sides corresponding angles direct proportion inverse proportion	°°Thales and the Pyramid See: B - p. 373
direct variation constant of variation inverse variation	

Unit VIII - Roots, Powers and Radicals (2 weeks)

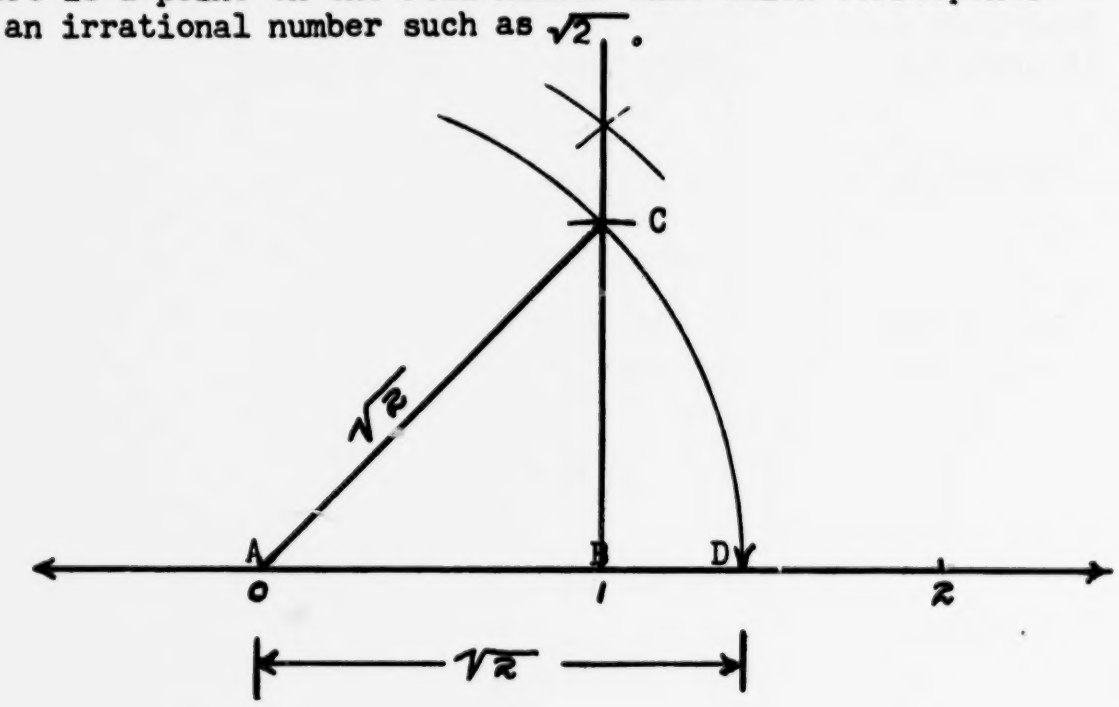
Topics	Objectives	References A	B	C	D	E
Square Root	To develop an understanding of the concept of square root	433	407-428	344-359	425-454 458-459	397-420
Powers and Radicals	To develop the ability to use the notation and vocabulary of powers and radicals	447-448				
	To develop the ability to use ° tables of powers and roots	434-436 °521-527	°415	°424	°439	°541-542
	To develop the ability to extract square roots of any positive integer to any number of decimal places	436-438				
	To develop the ability to find the square roots of open expressions which are perfect squares					
Pythagorean Theorem	To develop an appreciation of the Pythagorean Theorem	439-440				

Vocabulary	Suggestions
square root principal square root radical $\sqrt{\ }$ index indices radicand involution evolution power fractional exponent	Every positive number has two square roots which are equal in absolute value but opposite in sign. $\sqrt{25} = +5$ $25^{\frac{1}{2}} = +5$ $-\sqrt{25} = -5$ Square root of 25 = ± 5 $\pm\sqrt{25} = \pm 5$
perfect square	Rational numbers like 9, 36, $\frac{64}{49}$ which are squares of rational numbers are called perfect squares. Mention that in order to extract the square root of a negative number, the number system must be extended to include imaginary numbers. °°°Decimal representation of rational numbers See: E - pp. 400-403 Modern Algebra First Course by Johnson, Lendsey, Slesnick -- pp. 214-218
Pythagorean Theorem hypotenuse	°°°Euclid and Theon of Alexandria on square root °°°Investigate proofs of Pythagorean Theorem

Unit VIII - Roots, Powers and Radicals

Topics	Objectives	References A	B	C	D	E
Irrational Numbers	To develop an appreciation of irrational numbers	444,370				420M
Fundamental Operations With Radicals	To develop the ability to perform the fundamental operations with radicals	440-446 448-449				

Vocabulary	Suggestions
irrational number	There is a point on the real number line which corresponds to an irrational number such as $\sqrt{2}$.



Given: $\overline{AB}$ whose measure of length is 1 unit
$\overline{BC} \perp \overline{AB}$

$$\overline{BC} = \overline{AB}$$

$$\overline{AC} = \sqrt{\overline{BC}^2 + \overline{AB}^2}$$

$$\overline{AC} = \sqrt{1^2 + 1^2}$$

$$\overline{AC} = \sqrt{1 + 1}$$

$$\overline{AC} = \sqrt{2}$$

With A as center, swing an arc with radius $\overline{AC} = \sqrt{2}$ intersecting the number line at a point to be called D.

Since $\overline{AC}$ and $\overline{AD}$ are radii of the same circle, then $\overline{AC} = \overline{AD}$ and $\overline{AD} = \sqrt{2}$.

simplify
rationalize

Unit IX - Quadratic Equations and Inequations (3 weeks)

Topics	Objectives	References A	B	C	D	E
Quadratic Equations and Inequations	To develop the ability to recognize quadratic equations and inequations	457-472	434-455 460-464	370-389	460-479 378-384	465-485
Factoring	To develop the ability to solve quadratic inequations by factoring					479-480
Completing the Square	To develop the ability to derive the solution set for quadratic equations by completing the square					
Formula	To develop the ability to derive and use the quadratic formula					
Graphing	To develop the ability to solve quadratic equations by graphing					353-354
	To develop the ability to solve quadratic inequations by graphing		452-454			479-484
Verbal Problems	To develop the ability to solve verbal problems using quadratic equations					

Vocabulary	Suggestions
quadratic equation second-degree equation quadratic inequation pure quadratic equation	°°°Report on spirals, golden rectangles. See: B - p. 434 Review solving quadratic equations by factoring. °°°Explore various methods of solving quadratic equations, such as the Arabic, Persian, Hindu, and European methods. See: B - pp. 460-464 Write quadratic equations from given roots. See: B - pp. 449-450 Write equations from tables. See: C - pp. 156-160
discriminant	Study discriminant, $b^2 - 4ac$, which aids in determining the character of the roots without actually finding the roots. See: E - pp. 477-478

Unit X - Numerical Trigonometry (2 weeks)

Topics	Objectives	References				
		A	B	C	D	E
Indirect Measurement	To develop the ability to measure distances indirectly by using trigonometric ratios	399-425 491-514	465-488	397-415	485-506	493-514
Tables	To develop the ability to use ° tables of trigonometric ratios	°405	°474	°425	°493	°543
Problem Solving	To develop the ability to solve problems using trigonometric ratios					

Vocabulary	Suggestions
trigonometry similar triangles right triangle side adjacent side opposite hypotenuse tangent ratio sine ratio cosine ratio angle of elevation angle of depression interpolation	Much of the geometry material was previously covered in the seventh grade. Deriving trigonometric ratios: Have pupils draw several similar triangles. Derive trigonometric ratios by comparing the ratios of the sides. °°°Interpolation is a process for determining trigonometric ratios not given in the table. sine 26° 10' = 0.4410 sine 26° 20' = 0.4436 If the value of 26° 15' is not given in the table, we use the following argument to find it: Since the angle 26° 15' is halfway between 26° 10' and 26° 20', then sine 26° 15' is halfway between 0.4410 and 0.4436. Hence sine 26° 15' = 0.4423. °°°Sound Film: <u>Sine Language</u> - (30 min.) Association Films 561 Hillgrove Avenue La Grange, Illinois (Use after school or in Math. Club) ##Trigonometric Tables and Involute Functions "Good Formulas" - 6 for $1.00 (minimum order of 6) Illinois Tool Works 2501 N. Keeler Avenue Chicago 39, Illinois

VOCABULARY

A

abscissa
absolute value
abstract number
addend
addition
additive identity element
additive inverse
algebraic expression
algebraic sentence
angle of elevation
angle of depression
arithmetic scale
associative property

B

base
binomial
binomial factor
braces
brackets

C

closed sentence
closure
coincide
common monomial factor
commutative property
complex fraction
constant
constant of variation
continued ratio
coordinate
corresponding angles
corresponding sides
cosine ratio
counting number

D

decimal fraction
degree
denominator
density
dependence
dependent
dependent variable
directed number
direction
direct proportion
direct variation
discriminant
disjoint
distance
distributive property
dividend
division
divisor

E

element
empty set
equal
equation
equivalent
equivalent equations
equivalent inequations
evaluate
evolution
exponent
expression
extremes

F

factor
finite
formula
fractional exponent
function

G

general number
graph

H

half-plane
horizontal axis
hypotenuse

I

improper fraction
inclusion
inconsistent
independent
independent variable
index
indices
inequation
infinite
integer
intercept
interpolation
intersect
intersection
inverse
inverse proportion
inverse variation
involution
irrational number

J, K

L

largest common monomial factor
least common denominator
like terms
line vector
literal coefficient
literal equation
literal factor
literal inequation
literal notation
literal number
locus

M

magnitude
means
member
minuend
mixed decimal
mixed number
monomial
multiplicand
multiplication
multiplicative identity element
multiplicative inverse
multiplier

N

natural number
negative number
null set
number
number line
numeral
numerator
numerical coefficient
numerical factor

O

one
open expression
open sentence
opposite quality
order property
ordered pair
ordinate
origin

P

parallel
parentheses
perfect square
plane
point
polynomial
positive common fraction
positive number
power
prime factor
principal square root
proper fraction
proportion
pure decimal
pure quadratic equation
Pythagorean Theorem

Q

quadratic equation
quadratic inequation
quadrant
quotient

R

radical
radicand
ratio
rational number
rationalize
real number
reciprocal
reflexive property
relationship
replacement set
right triangle

S

scale drawing
scientific notation
sentence
set
set-builder
set notation
side adjacent
side opposite
signed number
similar terms
similar triangles
simplify
simultaneous equations
simultaneous inequations
slope
square root
standard form
subset
subtraction
subtrahend
sum
symmetric property

T

table
tabulate
tangent ratio
term
transitive property
trigonometry
trinomial

U

union
universal set
universe
unlike terms

V

variable
Venn diagram
vertical axis
vinculum

W

X

x-axis
x-intercept

Y

y-axis
y-intercept

Z

zero

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